ONLINE-APPENDIX

Unraveling Over Time

Sandro Ambuehl and Vivienne Groves

Part A of this Online Appendix includes the proofs to all propositions. It also shows that the assumptions placed on the conditional expectation functions in Section 3 are satisfied if the evolution of student skill levels follow independent Brownian motions. Part B contains three additional results, each developed in the case of two employers: (i) an analysis of the model in which the equilibrium is obtained as the limit of discrete time games, (ii) an extension of the model in which students can reject offers from employers, (iii) an extension of the model in which employers have full information about the current level of all students' skills at any point in time. Part C describes our data collection on the market for judicial law clerks.

Table of Contents

A Mathematical appendix

A.1 Proof that plays are well-defined

Consider an arbitrary strategy profile $\tilde{\sigma}$. We need to show that for each subgame, $\tilde{\sigma}$ maps into a uniquely defined payoff for each player. Hence, consider an arbitrary subgame. Let k denote the number of employers who have already left the market, and let $t_1 \leq \ldots \leq t_k$ denote the points in time at which they have left. Let $\tau_k \geq t_k$ denote the first point in time at which one of the remaining employers plays an action $e \in E$ in strategy profile $\tilde{\sigma}$. Crucially, because the strategies are required to be left-closed, this minimal time exists (if strategies were open on the left, only the infimum would exist, but not the minimum). Let N_k denote the subset of employers play some action $e \in E$ at τ_k. By construction, $|N_k| > 1$. Moreover, let $\hat{j}_k$ denote the maximal index j of actions e_j that are played at τ_k by one of the employers in N_k (that is, $\hat{j}_k$ is the rank of the worst student that one of the employers in N_k would still be willing to hire). At τ_k, therefore the number $\min\{|N_k|, \hat{j}_1\}$ employers and equally many students leave the market. These students and employers are matched as specified in Section 3; their payoffs are given by $x_j(\tau_k|t_1, \ldots, t_k, \tau_k, \ldots, \tau_k)$ for $k+1 \leq j \leq |N_k|$, with the assignment of these numbers to employers as determined by the matching at τ_k (recall that the function x_j has j arguments). Applying the same argument to the subgame starting at τ_k shows that the strategy profile $\tilde{\sigma}$ maps into uniquely defined payoffs for all market participants, as needed. $\square$

A.2 Proof of Proposition 1

For easier readability, we use the following notation. Consider a history H_t in which k employers have already exited at times $s_1, \ldots, s_k$. We then write

$$\phi_{k+1}(s_1, \ldots, s_k; t) = \inf\{t' \geq t | t' \text{ satisfies the properties in definition 1}\}$$

Moreover, we write

$$g_{k+1}(s_1, \ldots, s_{k-1}; t) = x_{k+1}(\phi_{k+1}(s_1, \ldots, s_{k-1}; t)|s_1, \ldots, s_{k-1}, t) \tag{1}$$

Intuitively, $g_{k+1}(s_1,\ldots,s_{k-1};t)$ is the value of remaining in the game when the first $k-1$ employers have exited at times $s_1,\ldots,s_{k-1}$, and the kth employer exits at time t.

We use induction over k, starting from $k=J$, to prove the following lemma

Lemma 1. *For any $k=1,\ldots,J$ and for any history H_t in which $k-1$ employers have exited at previous times $h_t=(s_1,\ldots,s_{k-1})$,*

(i) there exits a unique next candidate hiring time $\phi_k(s_1,\ldots,s_{k-1};t)$.

(ii) $g_{k+1}(s_1,\ldots,s_{k-1};t)$ is weakly decreasing in t, and is weakly decreasing in $s_1,\ldots,s_{k-1}$ for all $s_{k-1}\le t$.

To obtain the entire vector of future candidate hiring times associated with a given history H_t with previous hiring times $s_1,\ldots,s_{k-1}$ we then define recursively:

$$\begin{aligned} t_k(H_t) &= \phi_k(s_1,\ldots,s_{k-1};t) \\ t_{k+j}(H_t) &= \phi_{k+j}\big(s_1,\ldots,s_{k-1},t_k(H_t),\ldots,t_{k+j-1}(H_t)\big) \end{aligned}$$

thus finishing the proof.

To prove lemma 1, we first prove another lemma. Intuitively, in the case of $n=1$, lemma 2 states the following: Suppose f is a strictly increasing function of t, and g is a weakly decreasing function of t, and both f and g are continuous and cross at some point t'. Now shift both f and g upwards to some $\hat{f}$ and $\hat{g}$ such that $\hat{f}$ is strictly increasing in t, $\hat{g}$ is weakly decreasing in t, and $\hat{f}$ and $\hat{g}$ intersect at t''. Then $\hat{f}(t'')\ge f(t')$.

Lemma 2. *Let $\underline{t},\tilde{t},\bar{t}\in\mathbb{R}$ with $\underline{t}\le\tilde{t}\le\bar{t}$. Let $\Theta=\{(\theta_1,\ldots,\theta_n)\in[\underline{t},\tilde{t}]^n|\theta_1\le\theta_2\le\ldots\le\theta_n\}$. Let $f,g:[\underline{t},\bar{t}]\times\Theta\to\mathbb{R}$ be continuous functions. Suppose that both f and g are weakly decreasing in the second argument, θ, that f is strictly increasing, and that g is weakly decreasing in the first argument. Moreover, suppose that $f(\bar{t},\theta)>g(\bar{t},\theta)$ for all θ. Define $t^*(\theta)$ by*

$$f(t^*(\theta),\theta)=g(t^*(\theta),\theta) \tag{2}$$

if such a value exists and weakly exceeds $\tilde{t}$, and define $t^*(\theta)=\tilde{t}$ otherwise. Then

(i) $f(t^*(\theta),\theta)$ is weakly decreasing in θ,

(ii) $t^*(\theta)$ *is continuous.*

Proof.

(i) For all $\theta \in \Theta$, let $G(\theta) = \{(x,t) \in \mathbb{R} \times [t, \bar{t}] : x \leq \min\{f(t,\theta), g(t,\theta)\}\}$. If $f(t^*(\theta),\theta) = g(t^*(\theta),\theta)$, then because f is increasing and g is decreasing in its first argument, $f(t^*(\theta),\theta) = \max_{(x,t)\in G(\theta)} x$. If $t^*(\theta) = t$, then $f(t',\theta) \geq g(t',\theta)$ for all $t' \geq t$. Therefore, and because g is weakly decreasing in its first argument, we again have $f(t^*(\theta),\theta) = \max_{(x,t)\in G(\theta)} x$. Because both f and g are uniformly nonincreasing in the second argument, so is $\max_{(x,t)\in G(\theta)} x$, which completes the proof.

(ii) Let S denote the set of all θ such that a solution to (2) exists. $t^*(\theta)$ is continuous on the interior of S since f and g are both continuous, and it is trivially continuous on the complement of S. It remains to show that it is continuous on the boundary of S. For this it is sufficient to show that $t^*(\theta) = \tilde{t}$ for all θ on the boundary of S. To do so, define $\psi(t,\theta) := f(t,\theta) - g(t,\theta)$. Note that ψ is continuous and strictly increasing in its first argument. Set $\mathcal{T} := \psi^{-1}(\{0\})$. Since ψ is continuous, and since $\mathcal{T}$ is the preimage of a closed set, $\mathcal{T}$ is closed. Consequently, S, which is the projection of $\mathcal{T}$ to Θ, is a closed set. Now choose $\tilde{\theta}$ on the boundary of S. Suppose that $t^*(\tilde{\theta}) > \tilde{t}$. Then, due to monotonicity of ψ in the first argument, $\psi(\tilde{t},\tilde{\theta}) < 0$. Hence, by continuity of ψ, there exists $\delta > 0$ and $\eta > 0$ such that for all θ in an η-neighborhood of $\tilde{\theta}$ we have $\psi(\tilde{t}+\delta,\theta) < 0$. Hence, by continuity of ψ, and because $\psi(\bar{t},\theta) > 0$, we have that for all θ in the η-neighborhood of $\tilde{\theta}$, a solution to (2) exists. This contradicts the assumption that $\tilde{\theta}$ was chosen on the boundary of S.

□

Proof. (of lemma 1)

Induction anchor.

(i) Let H_t be a history in which $J-1$ employers have exited at times $s_1, \ldots, s_{J-1}$. Set $\phi_J(s_1, \ldots, s_{J-1}) = T$. Trivially, this is unique and satisfies the properties in definition 1.

(ii) Observe that by the above, $g_J(s_1, \dots, s_{J-2}; t) = x_J(T|s_1, \dots, s_{J-2}, t)$, which is jointly continuous and decreasing in all its arguments by the assumed properties of the conditional expectation functions.

Induction step.

Suppose lemma 1 holds for all histories H'_t in which k employers have exited. (This is the induction assumption.) We show that it then holds for all histories H_t in which $k-1$ employers have exited.

(i) Choose a history H_t in which $k-1$ employers have exited, and let $s_1, \dots, s_{k-1}$ denote the points in time at which they exited. We consider two cases.

Case 1

Suppose for all $t' \geq t$ we have
$x_k(t'|s_1, \dots, s_{k-1}) \neq x_{k+1}(\phi_{k+1}(s_1, \dots, s_{k-1}, t)|s_1, \dots, s_{k-1}, t)$. Part (iii) of definition 1 implies $\phi_k = t$. This shows uniqueness. For existence, it remains to show that $\phi_k = t$ satisfies the remaining properties of definition 1. (i) does not apply. If (iv) applies, then property (ii) follows directly by property (v) of the family of conditional expectation functions. If (iv) does not apply, then we need to show that (ii) is satisfied. Suppose this is not the case. Hence, $x_k(\phi_k|s_1, \dots, s_{k-1}) < x_{k+1}(\phi_{k+1}(s_1, \dots, s_{k-1}, \phi_k)|s_1, \dots, s_{k-1}, \phi_k)$. By property (i) of the family of conditional expectation functions, both sides of this inequality are continuous. By property (v), $x_k(T|s_1, \dots, s_{k-1}) > x_{k+1}(T(s_1, \dots, s_{k-1}, T)|s_1, \dots, s_{k-1},$ Hence, by the intermediate value theorem, there exists $t' \in (t, T)$ such that $x_k(t'|s_1, \dots, s_{k-1}) = x_{k+1}(t'|s_1, \dots, s_{k-1}, t')$, in contradiction to the assumption of this case.

Case 2

Suppose there is some $t' \geq t$ such that

$$x_k(t'|s_1, \dots, s_{k-1}) = x_{k+1}(\phi_{k+1}(s_1, \dots, s_{k-1}, t')|s_1, \dots, s_{k-1}, t'). \tag{3}$$

Observe that the right hand side of equation (3) is $g_{k+1}(s_1, \dots, s_{k-1}; t')$. Since the left hand side of equation (3) is strictly increasing in t' by property (ii) of the family of conditional expectation functions, and the right hand side is weakly decreasing in t' by the induction assumption, t' is the unique value that satisfies (3). By monotonicity, part (ii) of definition 1 precludes $\phi_j < t'$, and therefore, part (iii) of that definition requires $\phi_j = t'$. This shows uniqueness. For existence, we again show that ϕ_j satisfies parts (i) - (iv) of definition 1. (i) does not apply, and (ii) and (iii) are satisfied due to $\phi_j = t'$. Finally, (iv) does not apply due to property (v).

(ii) As just shown, if there exists $t' \geq t$ such that

$$x_k(t'|s_1, \ldots, s_{k-1}) = g_{k+1}(s_1, \ldots, s_{k-1}; t'), \tag{4}$$

then $\phi_k(s_1, \ldots, s_{k-1}; t) = t'$, and $\phi_k(s_1, \ldots, s_{k-1}; t) = t$ otherwise.

Observe that by the induction assumption, $t \geq s_{k-1}$. Therefore, $g_k(s_1, \ldots, s_{k-2}; s_{k-1}) = x_k(\phi_k(s_1, \ldots, s_{k-1}; s_{k-1})|s_1, \ldots, s_{k-1})$. Hence, a direct application of lemma 2 yields that $g_k(s_1, \ldots, s_{k-2}; s_{k-1})$ is continuous and weakly decreasing in all arguments.

□

A.3 Proof of Proposition 2

We prove this proposition permitting mixed strategies, defined as follows. A *mixed strategy* of an employer j is a function $\sigma_j : \mathcal{H} \to E \cup \{s\}$ that maps a history H_t into a cumulative probability distribution $F^j(\cdot; H_t) \in [0,1]^I$ with support contained in $[t, T]$. The ith component of $F^j(t'; H_t)$ is the probability that employer j plays one of the actions $e_1, \ldots, e_i$ in the interval $[t, t']$ if no other employers have taken any action $e \in E$ in the interval $[t, t')$. Because F^j is a cumulative distribution function, it is right-continuous. This implies that for any pure strategy an employer might have, in any subgame there is a well-defined first point in time at which an employer takes an action $e \in E$.[47]

We require each strategy σ_j to be internally consistent in the following sense: fix a history H_t, and define, for all $\tilde{t} \geq t$, the history $\tilde{H}_{\tilde{t}}$ as the history which coincides with H_t up to and including time t, and in which every employer plays action s at each point in time $[t, \tilde{t}]$. Then for any $t' \in [\tilde{t}, T]$, the probability that $\sigma_j(\tilde{H}_{\tilde{t}})$ assigns to an action $e \in E$ in time interval $[\tilde{t}, t']$ coincides with the probability that $\sigma_j(H_t)$ assigns to the same action e in the

[47] For instance, a plan of action in which an employer plays s for all $t \leq \hat{t}$ for some $\hat{t}$, and plays e_1 for all $t > \hat{t}$, is not an admissible strategy.

same time interval $[\tilde{t}, t']$ (see, e.g., Fudenberg, Tirole, "Preemption and rent equalization in the adoption of new technology," The Review of Economic Studies, 1985).

We now prove the two parts of Proposition 2.

(i) We show that σ is an SPE by arguing that no player has an incentive to unilaterally deviate to any other strategy.

Fix an arbitrary history H_t in which k employers have exited, and let $s_1, \ldots, s_{k-1}$ be the times of exit. Fix an arbitrary employer j who has not yet exited in history H_t. Define for all $t' > t$

$$y_j(t'; H_t) \quad := \quad x\Big(t' \Big| s_1, \ldots, s_{k-1}, \big(t_j(H_t)\big)_{t' > t_j(H_t), j \geq k}\Big)$$

Note that $\big(t_j(H_t)\big)_{t' > t_j(H_t), j \geq k}$ is the vector of all hiring times for history H_t that lie in the interval $[s_{k-1}, t')$.

Intuitively, $y_j(t'; H_t)$ is the payoff (student skill) that employer j obtains from taking an action that causes him to exit at time t', given the remaining employers' strategies.[48]

We prove the proposition in two steps.

First we show that other employers react to a deviation by employer j only in immaterial ways. Regardless of the strategy that employer j adopts, if, conditional on the other players' strategies σ_{-j}, this strategy makes employer j exit at time t', then j's expected payoff from this strategy is at most $y(t', H_t)$. (This is not a priori obvious, since a failure by employer j to exit at the point in time at which he would exit under σ_j could, in principle, lead to subgames in which other employers behave in a way that increases employer j's expected payoff.) Second, we show that the shape of y is such that employer j does not have any incentive to deviate to any other strategy, including mixed strategies.

Lemma 3. *Fix a history H_t. Suppose employer j deviates to a continuation strategy σ'_j. Let $F_j : [t, T] \to [0, 1]$ be the distribution of the time at which employer j exits in*

[48]Observe that $y_j(t'; H_t)$ does not take into account that an employer more desirable than j may play e_1 at some candidate hiring time $t_j(H_t)$. In this case, employer j does not obtain $y_j(t_j(H_t); H_t)$ from playing an action that causes him to exit at $t_j(H_t)$, but instead obtains $\lim_{t' \searrow t_j(H_t)} y_j(t'; H_t)$, which is strictly smaller than $y_j(t_j(H_t); H_t)$ by property (v) of the family of conditional expectation functions. Hence, $y_j(t'; H_t)$ is an upper bound on the payoff that player j can obtain by exiting at time t'.

strategy profile (σ_j', σ_{-j}) *in the continuation game. Then, employer* j*'s expected payoff satisfies*

$$\pi_j(\sigma_j', \sigma_{-j}) \leq \int_t^T y(t'; H_t) dF_j(t').$$

Proof. The strategies in σ satisfy the following two properties by construction. First, for any history H_t, at each point in time t' at which there are $l \geq 1$ hiring times, at least $l+1$ employers take some action $e \in E$ such that l employers exit at time t', and no employer takes any action $e \in E$ at times t' that do not coincide with any candidate hiring time. Hence, exit times depend on employer j's strategy only through the candidate hiring times it induces. Second, candidate hiring times depend solely on the points in time at which previous employers have exited. Conditional on exit times, they do not depend on what action was taken by which employer. Hence, exit times depend on employer j's strategy only if j exits at a point in time that is not a candidate hiring time for the given history. Consequently, employer j's payoff from exiting at t' with probability 1 is given by $y(t'; H_t)$ if no more desirable employer exits simultaneously. If a more desirable employer exits simultaneously, then, by property (v) of the family of conditional expectation functions, employer j's payoff is strictly less than $y(t'; H_t)$. Hence, the claim follows by taking the expectation of $y(t'; H_t)$ with respect to F_j. □

Lemma 4.

(a) $y(t'; H_t)$ *has a relative maximum at each* $t_j(H_t)$ *with* $t_j(H_t) > t$ *and* $j \geq k$

(b) For all $j \geq k$ *with* $t_j(H_t) = t$*, we have*
$x\big(t_j(H_t) | s_1, \ldots, s_{k-1}, t_k(H_t), \ldots, t_{j-1}(H_t)\big) \geq \sup_{t'>t} y(t'; H_t)$

(c) $y(t'; H_t) = y(t''; H_t)$ *for all* $t', t'' \in \{t_j(H_t) > t : j \geq k\}$

Proof. (a) is an immediate implication of properties (ii) and (v) of the family of conditional expectation functions. (b) is due to part (iv) of the definition of candidate hiring times, definition 1, and (c) follows immediately by part (iii) of that definition. □

By lemma 3, analyzing the properties of y is sufficient for bounding the payoff from a given deviation strategy σ_j' from above. By parts (a) and (b) of lemma 4, any strategy

σ'_j whose support is not a subset of $\{t_k(H_t), \ldots, t_J(H_t)\}$ is strictly dominated by σ_j. If in strategy profile σ and after history H_t employer j exits at time t, then by part (b) of lemma 4, employer j has no incentive to deviate to any other strategy.[49] If under strategy profile σ and history H_t employer j exits at time $t_{j'}(H_t) > t$, then employer j obtains expected payoff $y(t_{j'}; H_t)$. Hence, by part (iii) of lemma 4, employer j has no incentive to deviate to any other strategy. This completes the proof.

(ii) Fix a (possibly mixed) SPE profile σ. We prove by induction that for any history H_t in which $k-1$ employers have exited, the kth employer exits at the candidate hiring time $t_k(H_t)$ for this history with probability 1. We anchor the induction at $k = J$, and step from k to $k-1$. (Hence, the induction assumption implies that for any history H_t in which $k-1$ employers have exited, all subsequent employers exit at the candidate hiring times $t_{k+1}(h_t), \ldots, t_J(h_t)$ for this history.)

To anchor the induction, suppose $k = J-1$. Then, because $x_J(t|h_t)$ is strictly increasing in t, in any SPE profile the Jth student must be hired at T, which is the Jth candidate hiring time for any history.

The meat of the argument lies in the induction step, which proceeds roughly as follows. Let H_t be a history in which $k-1$ employers have already exited up until, and including time t. We first show that no employer will exit strictly before the next candidate hiring time (since at any such point in time, remaining in the game is preferable to exiting). We then suppose that with some probability, the kth exit occurs strictly after $t_k(H_t)$. Since exiting is preferable to remaining in the game at all such points in time, employers must put full probability on exiting in the interval of time during which the probability that the kth employer has not yet exited is positive. This implies that the support of each employers' exit-distribution must have the same supremum $\bar{t} > t_k(H_t)$. We then show that some employer has an incentive to deviate to a strategy in which he will have exited with probability 1 at some point strictly before $\bar{t}$. (If the strategies are deterministic, this is the incentive to undercut the competing employers' hiring times.) This contradicts the fact that each employer's exit-distribution must have the same supremum. Therefore, we conclude that $\bar{t} = t_k(H_t)$, and thus, that at least one employer exits with probability 1 at candidate hiring time $t_k(H_t)$

[49] By definition, the current time t is a candidate hiring time only if exiting immediately is weakly better than remaining in the game.

Formally, fix a history H_t with $k-1$ previous hiring times $h_t = (t_1, \ldots, t_{k-1})$ with $k \leq J-1$.

First, we show that if $t > t_{k-1}$, then no employer may put positive probability on exiting in the time interval $(t, t_k(H_t))$. For any history H'_t with previous hiring times h'_t, let $g_k(t'|h'_t)$ denote an employer's expected payoff from remaining in the continuation game if all students are hired at candidate hiring times in the continuation game. Formally, this is defined by equation (1). As shown in lemma 1, $x(t'|h_t) - g_k(t'; h_t)$ is strictly monotonically increasing, and by definition of $t_k(H_t)$, $x(t_k(H_t)|h_t) = g_k(t_k(H_t); h_t)$. Hence, for all $t' < t_k(H_t)$, we have $x(t'|h_t) < g_k(t'; h_t)$, so that exiting at any such t' cannot be a best response for any employer.

Second, we show that at least one employer j must put probability 1 on exiting at $t_k(H_t)$.

For any employer j we let $F_j(t')$ denote the probability that employer j exits in the interval $[t, t']$. (When we subsequently alter some employer j's strategy, we will do so by altering F_j. Which actions the employer will take to achieve this is defined implicitly, from F_j and the remaining employers' strategies.) Moreover, let $\tilde{J}$ denote the set of employers who are still in the game at t.

Consider an arbitrary employer $j \in \tilde{J}$. Let $\tilde{Q}_j(t')$ denote the probability that any employer other than j exits in the time interval $[t, t']$. $\tilde{Q}_j$ is given by

$$\tilde{Q}_j(t') = 1 - \prod_{j' \in \tilde{J}, j' \neq j} (1 - F_{j'}(t'))$$

Note that $\tilde{Q}_j$ may be discontinuous.

If employer j and some other employer j' play e_1 at the same point in time t' with positive probability, the outcome for player j will depend on whether j' is more or less desirable than j. Thus, it is useful to define $\bar{Q}_j(t')$ as the probability that employer j has been preempted by a more desirable employer at or before time t'. This c.d.f may also be discontinuous.

$$\bar{Q}_j(t') = \left[1 - \prod_{j' \in \tilde{J}, j' < j} (1 - F_{j'}(t'))\right]$$

This allows us to define $Q_j(t')$ as the probability that employer j is preempted by any employer within the time interval $[t, t')$, or is preempted by a more desirable employer at time t', as follows:

$$Q_j(t') = \lim_{t'' \nearrow t'} \tilde{Q}(t'') + \left(\bar{Q}_j(t') - \lim_{t'' \nearrow t'} \bar{Q}_j(t'') \right)$$

As shown just above, no employer exits strictly before $t_k(H_t)$. So for all $t' < t_k(H_t)$ we have $\tilde{Q}_j(t') = 0$. If the probability that some employer exits at $t_k(H_t)$, which is the case if $\tilde{Q}_j(t_k(H_t)) = 1$, then the claim is proved. Hence, suppose $\tilde{Q}_j(t_k(H_t)) < 1$.

We now derive employer j's expected payoff (student skills) from exiting at $t' \in [0, T]$. The probability that employer j manages to be the next employer to exit at the given time t', equals the probability that no other employer exits in $[t, t')$ or no more desirable employer exits at t'. This is given by $(1 - Q_j(t'))$. In this case, employer j obtains expected payoff $x_k(t'|h_t)$. Instead, if j is preempted by another employer, the expected payoff from his continuation strategy depends on the point in time s at which he was preempted. By the induction assumption, students are hired at candidate hiring times in the continuation game, so that employer j's expected payoff in the continuation game is given by $g_k(s; h_t)$. $Q_j(t')$ is the probability that employer j will be preempted by some other employer in the interval $[t, t']$, and $\int_0^{t'} g_k(s; h_t) \frac{1}{Q_j(t')} dQ_j(s)$ is employer j's expected payoff conditional on this event. Summing up, employer j's expected payoff from playing e_1 at t' is given by

$$\psi_j(t') \quad = \quad \big(1 - Q_j(t')\big) x_k(t'|h_t) + Q_j(t') \int_0^{t'} g_k(s; h_t) \frac{1}{Q_j(t')} dQ_j(s) \tag{5}$$

Hence, employer j's expected payoff from a strategy with c.d.f. of playing e_1 given by $F_1^j(t')$ (and $F_i^j(t') = 0$ for all $i > 1$) is

$$\Psi_j(t) = \int_t^T \psi(t') dF_1^j(t') \tag{6}$$

Now observe that best-responding requires the following lemma. (For any cumulative distribution function G defined on $[t, T]$, we let $\mathrm{supp}(G)$ denote the support of G, and we let $\sup(\mathrm{supp}(G))$ denote the supremum of this set.)

Lemma 5. *For all $j', j'' \in \tilde{J}$ we have* $\sup(\text{supp}(F^{j'})) = \sup(\text{supp}(F^{j''}))$. *By extension, for all $j \in \tilde{J}$,*

$$\sup(\text{supp}(F^j)) = \sup(\text{supp}(Q_j)) \tag{7}$$

Proof. Suppose w.l.o.g. that $\sup(\text{supp}(F_{j'})) > \sup(\text{supp}(F_{j''}))$ for some $j', j'' \in \tilde{J}$. This means that employer j' allocates positive probability mass to hiring the kth student to a time interval during which some other employer j'' has already hired that student with probability 1. At each time t' in this interval, $Q_j(t') = 1$, so that $\psi_j(t') = \int_0^{t'} g_k(s; h_t) dQ_j(s)$. This is strictly smaller than $x(t_k(H_t)|h_t)$, because for all $t' > t_k(H_t)$ we have $x(t'|h_t) > g_k(t'; h_t)$. Hence, employer j' could do strictly better by reallocating that probability mass to time $t_k(H_t)$. $\square$

Let $\bar{t} = \sup(\text{supp}(Q_j))$.

Lemma 6. *F^j is continuous at $\bar{t}$ for all employers $j \in \tilde{J}$.*

Proof. Recall that for all $t' > t_j(h_t)$ we have $x_k(t'|h_t) > x_k(t_k(H_t)|h_t) = g_k(t_k(H_t); h_t) > g_k(t'; h_t)$. Because $\frac{Q_j(\cdot)}{Q_j(t')}$ is a well-defined c.d.f., we thus obtain for all $t' > t_k(H_t)$:

$$x_k(t'|h_t) > \int_0^{t'} g_k(s; h_t) d\left(\frac{Q_j(s)}{Q_j(t')}\right) \tag{8}$$

By definition of $t_k(H_t)$, and because $Q_j(t') = 0$ for all $t' < t_k(H_t)$ (as shown previously) we have $\psi_j(t_k(H_t)) = x_k(t_k(H_t)|h_t)$.

For the least desirable remaining employer j, $Q_j(\bar{t}) = \tilde{Q}(\bar{t})$. By definition of $\bar{t}$, we have $Q_j(\bar{t}) = 1$, so that $\psi(\bar{t}) = \int_0^{\bar{t}} g_k(s; h_t) d\left(\frac{Q_j(s)}{Q_j(\bar{t})}\right)$. By the assumption that $\bar{t} > t_k(H_t)$, and by the fact that $x_k(t'|h_t) > g_k(t'; h_t)$, we derive that $\psi(t_k(H_t)) > \psi(\bar{t})$. Hence, F^j, which is a best response to Q_j, must not place any positive probability on $\bar{t}$, as such a strategy could be improved be reallocating that probability to $t_k(H_t)$ instead. Hence, because $\sup(\text{supp}(F^j)) = \bar{t}$, as previously derived, and because $\bar{t} > t_k(H_t)$, F^j must be continuous at $\bar{t}$ for the least desirable remaining employer $j \in \tilde{J}$. Therefore, for the next-to-least desirable remaining employer j', we obtain that $Q_{j'}(\bar{t}) = \tilde{H}(\bar{t})$, and can apply the same argument as above to show that $F^{j'}$ must be continuous at

$\bar{t}$. Inductively, we derive that F^j must be continuous at $\bar{t}$ for all remaining employers $j \in \tilde{J}$. $\square$

From lemma 5 we have that $\sup(\mathrm{supp}(F^j)) = \bar{t}$ for all j. Moreover, by lemma 6, $\mathrm{supp}(F^j)$ must contain $(\bar{t} - \delta, \bar{t})$ for some $\delta > 0$ (i.e. rightmost connected subset of $\mathrm{supp}(F^j)$ is a non-degenerate interval). Because F^j is a best response, its support must be a subset of maximizers t' of $\psi_j(t')$. Let $\hat{t}$ be the supremum of the set of these maximizers, i.e. $\hat{t} = \sup(\mathrm{argmax}_{t' \in [t, \bar{t}]} \psi(t'))$. Trivially, $\max_{t' \in [t, \bar{t}]} \psi_j(t') \geq \psi_j(t_j(h_t))$. Moreover, as shown before,
$\psi_j(t_j(h_t)) > \psi_j(\bar{t})$. By lemma 6, Q_j is continuous at $\bar{t}$ for all j, and therefore, ψ_j is continuous at $\bar{t}$. Because ψ_j is continuous at $\bar{t}$, and due to $\psi_j(t_j(h_t)) > \psi_j(\bar{t})$, we therefore conclude that there exists $\epsilon > 0$ such that $\psi_j(t') < \psi_j(t_j(h_t))$ for all $t' \in (\bar{t} - \min\{\epsilon, \delta\}, \bar{t})$. Consequently, $\sup(\mathrm{supp}(F^j)) \leq \hat{t} < \bar{t}$. This, however, contradicts lemma 5.

This finishes the proof of the induction step, and thus the proof of proposition 2.

We note that the above proof has the following immediate implications:

- No more than one employer can exit at a given hiring time $t_k(H_t) > t$. This is because if two employers exited, then the second would be exiting strictly before the next hiring time, in contradiction to what was just proved.
- Employers participate in rushes exactly as part (iv) of definition 1 requires: In case of a rush, there are simply multiple hiring times at the same time t.

A.4 Proof of Corollary 1

(i) This follows by construction of the subgame hiring times.

(ii) We prove this corollary by induction over j. To see the base case, suppose that $t'_1 \geq t_1$. By proposition 2, in equilibrium all employers obtain a student with the same expected skill level. In particular, $x_J(T|t'_1, \ldots, t'_{J-1}) = x_1(t'_1)$. By the assumption that $t'_1 \geq t_1$ and because $x_1(t)$ is strictly increasing, $x_1(t'_1) \geq x_1(t_1)$. Because $x_2(t|t_1)$ is strictly increasing in the first argument, and decreasing in the second, $x_2(t'_2|t'_1) = x_1(t'_1) \geq x_1(t_1) = x_2(t_2|t_1)$ implies $t'_2 \geq t_2$. Inductively we thus find that $t'_j \geq t_j$ for all $j \leq$

$J-1$. Because $x_J(T|t_1,\dots,t_{J-1})$ is decreasing in $t_1,\dots,t_{J-1}$, we thus obtain that $x_1(t_1') \geq x_J(T|t_1',\dots,t_{J-1}') > x_{J+1}(T|t_1',\dots,t_J')$. The last inequality is due to property (v) of the family of conditional expectation functions. This contradicts the fact that all hiring times yield the same expected student skill level.

For the induction step, suppose that $t_l' < t_l$ for all $1 \leq l \leq j-1$. Again, we use proof by contradiction. Assume $t_j' \geq t_j$. Then, $x_1(t_1') = x_j(t_j'|t_1',\dots,t_{j-1}') \geq x_j(t_j|t_1',\dots,t_{j-1}') \geq x_j(t_j|t_1,\dots,t_{j-1}) = x_1(t_1)$, where the equalities follow by proposition 2, the first inequality is because x_j is increasing in the first argument, and the second is due to the induction assumption and the fact that x_j is decreasing in all other arguments. This contradicts the fact that x_1 is strictly increasing.

(iii) We construct an equilibrium in which each student receives multiple simultaneous offers at each equilibrium hiring time, and in which the order of exit is $(j_1,\dots,j_{J-1},J)$ where $(j_1,\dots,j_{J-1})$ denotes an arbitrary permutation of the vector $(1,\dots,J-1)$. We alter part (iii) of the definition of equilibrium strategies as follows: $\sigma_j(H_t) = e_m$ if $t_{k+1}(H_t) = t_{k+m}(H_t) = t$ and if $x(t_{k+m}(H_t)|t_1,\dots,t_k,t_{k+1}(H_t),\dots,t_{k+m-1}(H_t))$ $> x(T|t_1,\dots,t_k,t_{k+1}(H_t),\dots,t_{J-1}(H_t))+\epsilon$. Each employer plays an equilibrium strategy with this alteration, and the ambiguity in part (iv) is resolved as follows: at the kth equilibrium hiring time, employers j_k and j_{k-1} play e_1. All other employers play s. At each hiring time that is off the equilibrium path, all employers play e_1. This strategy profile constitutes a subgame perfect ϵ equilibrium in which the order of exit on the equilibrium path is given by $(j_1,\dots,j_{J-1})$. Since this argument holds for all $\epsilon > 0$, it holds in the limit. The proof for the fact that a unique order of exit is consistent with exact equilibrium is shown in the main text.

A.5 Proof of Corollary 2

(i) This follows directly from the construction of the equilibria.

(ii) This is proved in the main text.

(iii) In the case of complements, the claim follows from the fact that an assortative match at T maximizes output. In the case of substitutes, we need to show the existence of an increasing vector of parameters a_j such that the unraveling equilibrium leads to smaller expected output than an assortative match at T. As shown in Subsection 4.4, the least desirable employer, employer J expects a higher student skill level in the unraveling equilibrium than from an assortative matching at T. Accordingly, if $a_j = 1$ for all $j \leq J-1$, then expected production in the unraveling equilibrium will be higher than that from an assortative match at T as a_J grows sufficiently large.

A.6 Proof of Proposition 3

To simplify notation, we summarize a vector of the form $(t_0, \ldots, t_0, T, \ldots, T)$ with k entries t_0 and k' entries T by (k, k'). We let $x_{k+1}(t_0|k) = x_{k+1}(t_0|t_0, \ldots, t_0)$, for $k, k' \in \{1, \ldots, J\}$. Hence, $x_{k+k'+1}(T|k, k') = x_{k+k'+1}(T|t_0, \ldots, t_0, T, \ldots, T)$, where the vector $t_0, \ldots, t_0, T, \ldots, T$ in the foregoing expression contains k entries t_0 and k' entries T.

(i) First, we show that any equilibrium strategy profile is in threshold strategies.

Fix an arbitrary pure strategy SPE profile σ.

We show that there is an employer $\hat{j} \in \{1, \ldots, J\}$ such that the $\hat{j}$ most desirable employers $j \leq \hat{j}$ exit at T, and the remaining employers exit at t_0. (Observe that this implies that expected equilibrium payoffs are weakly higher for more desirable employers, $\pi_1 > \cdots > \pi_{\hat{j}} = \pi_{\hat{j}+1} = \cdots = \pi_J$.)

Suppose this is not the case. We show that there exists an employer who has an incentive to deviate. Let k be the number of employers who hire at t_0. Consider the most desirable employer j_1 who hires at t_0. From hiring at t_0, this employer obtains payoff $z_{j_1}(t_0) = x_1(t_0) = 0$. If he deviated to hiring late, this employer would obtain $x_{k+j_1-1}(T|k-1, j_1-1)$. This is because if j_1 hires at T, $k-1$ students will have been hired at t_0, and $j_1 - 1$ more desirable employers will compete with j_1 at T (because j_1 is the most desirable employer who hires early). Because σ is an equilibrium profile, j_1 cannot profit from this deviation. Hence, $x_{k-1+j_1}(T|k-1, j_1-1) \leq x_1(t_0) = 0$. Now consider the best employer $j_2 > j_1$ who is less desirable than j_1 and who hires at T. By doing so, he obtains at most $x_{k+j_1}(T|k-1, j_1)$. By property (v) of the family of conditional expectation functions, $x_{k+j_1}(T|k, j_1-1) < x_{k-1+j_1}(T|k-1, j_1-1) \leq x_1(t_0) = 0$. By hiring at t_0 instead, j_2 can obtain 0.

It remains to show that a pure strategy equilibrium profile exists. For any $k \in \{0, \ldots, J-1\}$, define $y(k) = x_{k+1}(T|k, 0)$. This is the expected payoff of the least desirable employer who exits at T, when k employers exit at t_0. Because $x_J(t_0|J-1) = 0$, and because $x_J(t|J-1)$ is strictly increasing in t by property (i) of the family of conditional expectation functions, we have $y(J-1) = x_J(T|J-1, 0) > 0$. Hence, there exists $k \in \{0, \ldots, J-1\}$ such that $y(k)$ is nonnegative. Let k^* be the smallest such number. Then define the strategy profile σ as follows: Any sufficiently desirable employer $j \leq J - k^*$ exits at T and all remaining employers exit at t_0. It follows by the definition of $y(k^*)$ that no employer has an incentive to deviate.

Finally, it is immediate that if $x_J(T|J-1) > 0$, then the unique equilibrium is for all employers to exit at T.

(ii) To show that there is no pure strategy equilibrium if t_0 is sufficiently close to T, we first consider the case in which student qualities do not change over time. Fix a pure strategy profile σ. Let J_1 and J_2 be the sets of employers that hire at t_0 and T, respectively. For all $j = 1, \ldots, J$, let $y_j := x_j(T|0, j-1)$ be the payoff from hiring the jth best student at T. Then, the k-most desirable employer in set J_1 obtains expected payoff y_k. Similarly, the k'-most desirable employer in set J_2 obtains $y_{|J_1|+k'}$. Thus, any employer in J_1 earns at least as much as any employer in J_2.

In this case, there are at most two pure strategy equilibria: (1) all employers hire at t_0, and (2) all but the least desirable employer J hire at t_0, and employer J hires at T. We show that no other strategy profile can be an equilibrium.

Let $\hat{j}$ denote the least desirable employer who hires early in profile σ, and suppose $\hat{j} \leq J-2$. Then, employer $\hat{j}$ earns at least y_{J-2}. Moreover, employer J and $J-1$ hire at T, and earn y_J and y_{J-1}, respectively. By deviating to hiring at t_0, employer J can improve his payoff to at least y_{J-1}.

Second, we consider the case in which $t_0 < T$ so that student skills do change over time. Note that in the first case, we have derived strictly positive deviation incentives for all but two strategy profiles σ. Hence, because there are finitely many strategy profiles, and because the conditional expectation functions are continuous by property (i) of the family of conditional expectation functions, standard topological arguments show that there exists $\eta > 0$ such that for all $t_0 \in (T - \eta, T)$, for any strategy profile σ for which some player had an incentive to deviate when student skills do not change over time, there is some player who has an incentive to deviate when they do change.

Hence it remains to show that it neither is a pure strategy equilibrium for all employers to hire early, nor for all but the least attractive employer to hire early. If all employers hire at t_0, then employer J obtains $x_J(t_0|J-1)$. By deviating to hiring at T, employer J obtains $x_J(T|J-1,0)$ which is strictly profitable by property (i) of the family of conditional expectation functions. If all but the least attractive employers hire early, then employer $J-1$ obtains $x_{J-1}(t_0|J-2)$. By deviating to hiring at T, employer $J-1$ obtains $x_{J-1}(T|J-2,0)$ which is strictly profitable, again by property (i).

Thus, for all $t_0 \in (T-\eta, T)$ there is no pure strategy equilibrium.

Finally, we show that if t_0 is sufficiently close to T, then in any equilibrium, the employers' expected equilibrium payoffs satisfy $\pi_1 > \pi_2 > \ldots > \pi_J$. Consider any employer $k \in \{1, \ldots, J\}$. From hiring at t_0, employer k will obtain at least

$$\pi_k > x_k(t_0|k-1) \tag{9}$$

(since the previous expression is strictly decreasing in k by property (v) of the family of conditional expectation functions, and since $k-1$ employers are more desirable than employer k). This provides a lower bound on employer $\tilde{j}$'s expected equilibrium payoff.

Moreover, the sum of payoffs to employers cannot exceed π_{max}, defined by $\pi_{max} = \sum_{j=1}^{J} x_j(T|T, \ldots, T)$. Hence, we bound employer k's payoff from above by

$$\begin{aligned} \pi_k &< \pi_{max} - \sum_{j=1, j\neq k}^{J} x_j(t_0|j-1) \\ &= x_k(T|0, k-1) + \sum_{j=1, j\neq k}^{J} \left[x_j(T|0, j-1) - x_j(t_0|j-1)\right] \end{aligned} \tag{10}$$

As t_0 approaches T, the second term in expression (10) approaches 0, by continuity of x_j. Moreover, the left hand side of (9) approaches $x_k(T|0, k-1)$. Consequently, the payoffs in any equilibrium approach the payoffs from an assortative matching at T. Hence, there exists $\delta > 0$ such that for $t_0 \in (T-\delta, T)$ we have $\pi_1 > \pi_2 > \ldots > \pi_J$ in any equilibrium, as was to be shown.

Hence, part (ii) of proposition 3 holds for all $t_0 \in (T - \min\{\eta, \delta\}, T)$, which completes the proof.

A.7 Student skills given by Brownian motions

In this subsection, we show that a collection of Brownian motions $\left(q_i(t)\right)_{1\leq i\leq I}$ satisfies properties $(i)-(v)$ we assume for the conditional expectation functions $\left(x_j(t|t_1,\ldots,t_{j-1})\right)_{1\leq j\leq J}$. Stating the formal result requires notation. Let $(\Omega, \mathcal{F}, (\mathcal{F}_t)_t, P)$ be a filtered probability space. Let q be an I-dimensional Brownian motion defined on this space with $q(0)=0$. For path $\omega \in \Omega$ and time $t \in [0,T]$, we write $q^\omega(t) = (q_1^\omega(t), \ldots, q_I^\omega(t))$, and use W to denote the Wiener measure. For $x \in \mathbb{R}^I$, we let $\overline{\text{argmax}}(x)$ denote the minimal i such that $x_i \geq x_{i'}$ for all $i, i' \in \{1,\ldots,I\}$. We denote the set of available students after previous students have been picked at times $\tau_1,\ldots,\tau_k$ by $a_{k+1}(\tau_1,\ldots,\tau_k)$. We recursively define $a_{k+1}(\tau_1,\ldots,\tau_k)$ by setting $a_1 = \{1,\ldots,I\}$, and

$$a_{k+1}(\tau_1,\ldots,\tau_k) \quad = \quad a_k(\tau_1,\ldots,\tau_{k-1}) \setminus \overline{\text{argmax}}_{i\in a_k(\tau_1,\ldots,\tau_{k-1})} q_i(\tau_k)$$

Additionally, we define

$$i^*(t, a_{k+1}(\tau_1,\ldots,\tau_k)) = \overline{\text{argmax}}_{i\in a_k(\tau_1,\ldots,\tau_k)} q_i(t).$$

Note that both a and i^* are stochastic processes.

Before we state the formal proposition, we derive expressions for the conditional expectation functions. The conditional expectation function $x_{k+1}(t|\tau_1,\ldots,\tau_k)$ is given by

$$\begin{aligned} x_{k+1}(t|\tau_1,\ldots,\tau_k) &= E\left[q_{i^*(t,a_{k+1}(\tau))}(T)\right] \\ &= E\left[E\left[q_{i^*(t,a_{k+1}(\tau))}(T)|\mathcal{F}_t\right]\right] \\ &= E\left[E\left[\max_{i\in a_{k+1}(\tau)}\{q_i(t)\}\middle|\mathcal{F}_t\right]\right] \\ &= E\left[\max_{i\in a_{k+1}(\tau)}\{q_i(t)\}\right] \end{aligned}$$

The second and fourth line follow by the law of iterated expectations, the third by the martingale property of the Brownian motion. We now prove the following proposition.

Proposition 6. *If $\left(q_i(t)\right)_{1\leq i\leq I}$ is a family of independent standard Brownian motions with $q_i(0)=0$ for all $i \in \{1,\ldots,I\}$, then the conditional expectation functions $\left(x_j(t|t_1,\ldots,t_{j-1})\right)_{1\leq j\leq J}$ satisfy properties (i) to (v) of Section 3.*

To prove the proposition, we define further notation as follows. $\mathcal{N}(\mu, \sigma^2)$ denotes the unidimensional normal distribution with mean μ and variance σ^2. The symbol $\succsim$ denotes first order stochastic dominance, and $\simeq$ denotes equality in distribution. In the Euclidian space $\mathbb{R}^n$, $\geq$ denotes the product order[50] and $\mathbb{B}_\epsilon(x)$ denotes the ϵ-ball with radius ϵ and center x.

We now show that each of the properties (i) to (v) hold.

(i) To show that $x_{k+1}(t|\tau_1, \ldots, \tau_k)$ is jointly continuous in all arguments, choose $t \in [0, T]$ and $\tau \in [0, T]^k$, with $t \geq \tau_k \geq \ldots \geq \tau_1$. Fix a sequence $(t^l, \tau^l)_l$ that converges to (t, τ), such that $t^l \geq \tau_k^l \geq \ldots \geq \tau_1^l$ for all l. We need to show that $\lim_{l\to\infty} |x_{k+1}(t^l|\tau^l) - x_{k+1}(t|\tau)| = 0$.

We define

$$X_l = \max_{i \in a(\tau^l)} q_i(t^l) - \max_{i \in a(\tau)} q_i(t)$$

Note that $\left|x_{k+1}(t^l|\tau_1^l, \ldots, \tau_k^l) - x_{k+1}(t|\tau_1, \ldots, \tau_k)\right| = |E(X_l)|$. By Jensen's inequality, we have $|E(X_l)| \leq E(|X_l|)$. It thus suffices to show that $X_l \to 0$ in the $\mathcal{L}_1$-norm. We proceed in two steps. Lemma 7 below shows that X_l converges to 0 almost everywhere. Lemma 8 below shows that the sequence X_l is uniformly integrable. A direct application of the Lebesgue-Vitali theorem (theorem 4.5.4 in Bogachev (2007)[51]) then yields $X_l \to 0$ in $\mathcal{L}_1$.

Lemma 7. $P(\lim_{l\to\infty} X_l = 0) = 1$

Proof. Let $\Omega' = \{\omega \in \Omega : q(\omega) \text{ is continuous}\}$. It is a standard result in the theory of Brownian motions that $P(\Omega') = 1$. Let $\Omega'' = \{\omega \in \Omega : q_i(t') \neq q_{i'}(t')\ \forall t' \in \{\tau_1, \ldots, \tau_k, t\}\ \forall i \neq i'\}$. Because the Brownian motion has Gaussian increments, we have $P(\Omega'') = 1$, and, by implication, $P(\Omega' \cap \Omega'') = 1$.

Let $\omega \in \Omega' \cap \Omega''$. Because the path $q^\omega(t')$ is continuous in t', we get that for all $t' \in \{\tau_1, \ldots, \tau_k, t\}$ there exists $\epsilon(t') > 0$ such that for all $t'' \in \mathbb{B}_{\epsilon(t')}(t')$ we have $\overline{\operatorname{argmax}}_{i' \in a_k(\tau)} q_{i'}^\omega(t') = \overline{\operatorname{argmax}}_{i' \in a_k(\tau)} q_{i'}^\omega(t'')$.

[50] I.e. for $x, y \in \mathbb{R}^n$ we have $x \geq y \Leftrightarrow x_i \geq y_i\ \forall 1 \leq i \leq n$.
[51] Bogachev, V. I. (2007) *Measure Theory*, Springer, Berlin.

Because $(\tau^l, t) \to (\tau, t)$, we get that for all sufficiently large l, $a_k(\tau) = a_k(\tau^l)$ and $\overline{\text{argmax}}_{i\in a_k(\tau)} q_i(t) = \overline{\text{argmax}}_{i\in a_k(\tau)} q_i(t^l)$. Then it follows by continuity of q^ω that $q_{i^*(t,a(\tau))}(t) = \lim_l q_{i^*(t,a(\tau^l))}(t^l)$ as was to be shown. □

Lemma 8. *The sequence $(X_l)_l$ is uniformly integrable.*

Proof. For each l, let $(Y_i^l)_{i\in 1,\ldots,I}$ denote a sequence of vectors of $\mathcal{N}(0, |t - t^l|)$ random variables, and let $(Z_i^l)_{i\in 1,\ldots,I}$ denote a sequence of vectors of $\mathcal{N}(0, \min\{t, t^l\})$ random variables such that for each l, and for all $i \neq i'$, Y_i^l and Z_i^l are independent.

Let $\bar{u}_l = \max_{i\in I} |q_i(t^l)|$, and $u_l = \max_{i\in a(\tau^l)} q_i(t^l)$. Likewise, let $\bar{v}_l = \max_{i\in I} |q_i(t)|$, and $v_l = \max_{i\in a(\tau)} q_i(t)$. Trivially, $\bar{u}_l \geq u_l$, and $\bar{v}_l \geq v_l$.

We can bound $|X^l|$ as follows.

$$\begin{aligned} |X_l| &= |u_l - v_l| \\ &\leq |u_l| + |v_l| \\ &\leq \bar{u}_l + \bar{v}_l \\ &= \max_{i\in 1,\ldots,I} |Z_i^l + Y_i^l| + \max_{i\in 1,\ldots,I} |Z_i^l| \\ &\leq \max_{i\in 1,\ldots,I} |Y_i^l| + 2\max_{i\in 1,\ldots,I} |Z_i^l| \end{aligned} \tag{11}$$

The second and the last line follow by the triangle inequality, the fourth line is due to Jensen's inequality, and the penultimate line follows by the independent Gaussian increments property of the Brownian motion. By the above, it suffices to show that the sequence $\left(\max_{i\in 1,\ldots,I} |Y_i^l| + 2\max_{i\in 1,\ldots,I} |Z_i^l|\right)_l$ is uniformly integrable.

It is a standard result in probability theory that if Z is a $\mathcal{N}(0, \sigma^2)$ variable, then $E(Z|Z \geq c) = \sigma\frac{\phi(c)}{1-\Phi(c)}$, where ϕ and Φ denote the p.d.f. and c.d.f. of the standard normal distribution, respectively. Thus, $E(|Z|\mathbb{1}_{\{|Z|\geq c\}}) = 2E(Z|Z \geq c)P(Z \geq c) = 2\sigma\phi(c)$.

Note that

$$\begin{aligned} E\left(\left(\max_{i\in 1,\ldots I} |Y_i^l|\right)\mathbb{1}_{\{\max_{i\in 1,\ldots I} |Y_i^l|\geq c\}}\right) &\leq \sum_{i\in 1,\ldots I} E\left(|Y_i^l|\mathbb{1}_{\{|Y_i^l|\geq c\}}\right) \\ &= 2I|t - t^l|\phi(c) \end{aligned}$$

Similarly, $E\left(\left(\max_{i\in 1,\dots,I}|Z_i^l|\right)\mathbb{1}_{\{\max_{i\in 1,\dots,I}|Z_i^l|\geq c\}}\right)\leq 2I|\max\{t,t^l\}|\phi(c)$

Combining these results with (11), we can derive the following bound

$$\begin{aligned}\int_c^\infty |X_l|dW &\leq 2\int_c^\infty \max_{i\in 1,\dots I}|Y_i^l|dF_Y + 4\int_c^\infty \max_{i\in 1,\dots I}|Z_i^l|dF_Z \\ &\leq \left(2I|t-t^l| + 4I\max\{t,t^l\}\right)\phi(c)\end{aligned}$$

where F_Y and F_z denote the distributions of the random vectors Y and Z, respectively. Because $t^l \to t$, both sequences $\left(|t-t^l|\right)_l$ and $\left(\max\{t,t^l\}\right)_l$ are bounded above , and hence, the sequence $\left(2I|t-t^l| + 4I\max\{t,t^l\}\right)_l$ is bounded above by some $C\in\mathbb{R}$. Therefore,

$$\lim_{c\to\infty}\sup_l \int_c^\infty |X_l|dW \leq \lim_{c\to\infty} C\phi(c) = 0$$

as was to be shown.

□

(ii) To show that $x_{k+1}(t|\tau_1,\dots,\tau_k)$ is strictly increasing in t, fix $\tau\in[0,T]^k$ with $\tau_k\geq\dots\geq\tau_1$, and let $t'>t\geq\tau_k$. By the law of iterated expectations,

$$\begin{aligned}x_{k+1}(t'|\tau) &= P\left(i^*(t',a(\tau)) = i^*(t,a(\tau))\right)\cdot \\ &\quad E\left[\max\{q_i(t'):i\in a(\tau)\}|i^*(t',a(\tau)) = i^*(t,a(\tau))\right] \\ &\quad +P\left(i^*(t',a(\tau)) \neq i^*(t,a(\tau))\right)\cdot \\ &\quad E\left[\max\{q_i(t'):i\in a(\tau)\}|i^*(t',a(\tau)) \neq i^*(t,a(\tau))\right]\end{aligned}$$

Now, $E\big[\max\{q_i(t'):i\in a(\tau)\}|i^*(t',a(\tau)) = i^*(t,a(\tau))\big] =$ $E\left[\max\{q_i(t):i\in a(\tau)\}|i^*(t',a(\tau)) = i^*(t,a(\tau))\right]$ by the martingale property and the conditioning. Moreover, $E\big[\max\{q_i(t'):i\in a(\tau)\}|i^*(t',a(\tau))$ $\neq i^*(t,a(\tau))\big] > E\left[\max\{q_i(t):i\in a(\tau)\}|i^*(t',a(\tau)) \neq i^*(t,a(\tau))\right]$ by the definition of i^*. Finally, $P\left(i^*(t',a(\tau)) \neq i^*(t,a(\tau))\right) > 0$ because the Brownian motion has Gaussian increments and thus, $q(t')|q(t)$ has full support on $\mathbb{R}^I$. Therefore,

$$\begin{aligned} x_{k+1}(t'|\tau) &> P\left(i^*(t',a(\tau)) = i^*(t,a(\tau))\right) \cdot \\ &\quad E\left[\max\{q_i(t) : i \in a(\tau)\} | i^*(t',a(\tau)) = i^*(t,a(\tau))\right] \\ &\quad +P\left(i^*(t',a(\tau)) \neq i^*(t,a(\tau))\right) \cdot \\ &\quad E\left[\max\{q_i(t) : i \in a(\tau)\} | i^*(t',a(\tau)) \neq i^*(t,a(\tau))\right] \\ &= x_{k+1}(t|\tau) \end{aligned}$$

as was to be shown.

(iii) We prove two lemmas that imply the claim. For each $d \in \{1,\dots,I\}$, we define the following three mappings. $\varphi_d : \mathbb{R}^d \to \mathbb{R}^{d-1}$ is given by $\varphi_d(x) = (x_1,\dots,x_{i^*(x)-1}, x_{i^*(x)+1},\dots,x_d)$. Hence, $\varphi_d(x)$ equals x with the highest element removed. For any $1 \le k \le d$, we define $\pi_d^k : \mathbb{R}^d \to \mathbb{R}^{d-1}$ by $\pi_d^k(x) = (x_1,\dots,x_{k-1},x_{k+1},\dots,x_d)$. Finally, $\psi_d : \mathbb{R}^d \to \mathbb{R}^d$ is the mapping that orders a vector by size, largest dimension first (lowest index first in case of ties). Plainly, for any random vector X, the order statistics of X and $\psi_d(X)$ are identical.

Intuitively, the following lemma shows that when employer j' picks before some time t', the pool of remaining students at t' is better (in the f.o.s.d. sense) than if he picks at t'.

Lemma 9. *Let X and Y be independent $\mathbb{R}^d$-valued random variables. Let $M = \overline{\operatorname{argmax}}_{1\le i\le d} X_i$. Then,*

$$\psi_d\left[\varphi_d(X) + \pi_d^M(Y)\right] \succsim \psi_d\left[\varphi_d(X+Y)\right] \tag{12}$$

Proof. Let $U = X + Y$. Let R denote the ranking of U_M. I.e. $R = 1$ if U_M is the highest element of U, $R = 2$ if U_M is the second highest element of U, and so on. Then, letting LH and RH denote the left and right hand sides of equation (12), respectively, we get

$$\begin{aligned} LH &= \left[\left(\psi_d(U)\right)_1,\dots,\left(\psi_d(U)\right)_{R-1},\left(\psi_d(U)\right)_{R+1},\dots,\left(\psi_d(U)\right)_d\right] \\ RH &= \left[\left(\psi_d(U)\right)_2,\dots,\left(\psi_d(U)\right)_R,\left(\psi_d(U)\right)_{R+1},\dots,\left(\psi_d(U)\right)_d\right] \end{aligned}$$

Hence, $LH_i \geq RH_i$ for all $i \leq R-1$, and $LH_i = RH_i$ for all $i \geq R$. Consequently, $P(LH \geq RH) = 1$ so that the claim is a direct application of Strassen's theorem (theorem 2.4 in chapter IV in Lindvall, T., *Lectures on the Coupling Method*, New York, 2002). $\square$

Intuitively, the following lemma shows that if the best student i is picked from a pool of students, then the better this pool was before the best student was picked (in the f.o.s.d. sense), the better the pool of the remaining students (in the f.o.s.d. sense).

Lemma 10. *Let X, X', and Y be $\mathbb{R}^d$-valued random variables, such that*

(a) $X \succsim X'$

(b) X and X' are both independent of Y.

Then, $\varphi_d(X+Y) \succsim \varphi_d(X'+Y)$.

Proof. By Strassen's theorem, the stochastic dominance relation $X \succsim X'$ is equivalent to the existence of a coupling $\hat{P}$ such that $\hat{P}(X \geq X') = 1$.[52] Define the random variable (Y_1, Y_2) on $\mathbb{R}^d \times \mathbb{R}^d$ such $Y_1 \simeq Y$, $Y_2 \simeq Y$, and $\hat{P}(Y_1 = Y_2) = 1$. Then, $\hat{P}(X + Y \geq X' + Y) = 1$, and by monotonicity of φ_d we obtain $\hat{P}\left(\varphi_d(X+Y) \geq \varphi_d(X'+Y)\right) = 1$. By Strassen's theorem this is equivalent to the claim. $\square$

We now combine lemmas 9 and 10 to show that $x_{k+1}(t|t_1, \dots, t_k)$ is decreasing in t_j for all $j \in \{1, \dots, k\}$. Fix $t_1 \leq \ldots \leq t_k$, and fix some $1 \leq j \leq k$. Then, if $j > 1$, let $t'_j \in (t_{j-1}, t_j)$. If instead $j = 1$, let $t'_j \in [0, t_j)$. (For $t'_j < t_{j-1}$, the claim follows by repeated application of the current argument.)

For any subset $a \subseteq \{1, \dots, I\}$, let $\rho^a : \mathbb{R}^I \to \mathbb{R}^{|a|}$ be given by $\rho^a(x) = (x_i)_{i \in a}$. Let X be a random variable on $\mathbb{R}^{I-j}$ with $X \simeq \rho^{a(t_1,\dots,t_j)}\big(q(t'_j)\big)$. Let Y be an $(I-j)$-dimensional normal variable with mean zero and covariance matrix $(t_j - t'_j)\mathbb{I}_{I-j}$ that is independent of X, where $\mathbb{I}_d$ is the identity matrix in $\mathbb{R}^d$. Because the Brownian motion has independent Gaussian increments, $\rho^{a(t_1,\dots,t_j)}\big(q(t_j)\big) \simeq X + Y$.

[52] A coupling of two probability measures P and P' on the same measurable space $(E, \mathcal{E})$ is any probability measure $\hat{P}$ on the product measurable space $(E \times E, \mathcal{E} \otimes \mathcal{E})$ (where $\mathcal{E} \otimes \mathcal{E}$ is the smallest σ-field containing $\mathcal{E} \times \mathcal{E}$) whose marginals are P and P'.

If the j'th student is picked at t_j, the distribution of the order statistics of the remaining students at t_j is given by $\psi_{I-j}\big(\varphi_{I-j+1}(X+Y)\big)$. If the j'th student is picked at t'_j instead, the distribution of the order statistics of the remaining students at t_j is given by $\psi_{I-j}\big(\varphi_{I-j+1}(X)+\pi^M_{I-j+1}(Y)\big)$, where $M=\overline{\operatorname{argmax}}_i X_i$.

By lemma 9 we thus have that

$$\psi_{I-j}\left(\rho^{a(t_1,\ldots,t_{j-1},t'_j)}\big(q(t_j)\big)\right)\succsim\psi_{I-j}\left(\rho^{a(t_1,\ldots,t_{j-1},t_j)}\big(q(t_j)\big)\right)$$

The above shows that the distribution of the quality of the remaining students at t_j is better if the j'th student is picked at t'_j than if he is picked at t_j.[53] By iterated application of lemma 10 and the fact that the Brownian motion has independent Gaussian increments it now follows that this relation survives to time t in spite of the fact that at times $t_{j+1},\ldots,t_k$, the best available students are picked off the market. Formally, we obtain

$$\psi_{I-j}\left(\rho^{a(t_1,\ldots,t_{j-1},t'_j,t_{j+1},\ldots,t_k)}\big(q(t)\big)\right)\succsim\psi_{I-j}\left(\rho^{a(t_1,\ldots,t_{j-1},t_j,t_{j+1},\ldots,t_k)}\big(q(t)\big)\right)$$

Because the maximum is an increasing function, it thus follows from the definition of first order stochastic dominance that

$$E\left[\max_{i\in a(t_1,\ldots,t_{j-1},t'_j,t_{j+1},\ldots,t_k)}q_i(t)\right]\geq E\left[\max_{i\in a(t_1,\ldots,t_{j-1},t_j,t_{j+1},\ldots,t_k)}q_i(t)\right]$$

as was to be shown.

(iv) Let $q_{(i)}(t)$ denote the ith highest dimension of $q(t)$. Because the increments of the Brownian motion have full support on $\mathbb{R}^I$,
$P\left(q_i(T)<q_{(I-J)}(T)|q_i(t)=q_{(I)}(t)\right)>0$ for all $t<T$. Hence,
$\sum_j x_j(T|T,\ldots,T)>\sum_j x_j(t_j|t_1,\ldots,t_{j-1})$, as was to be shown.

(v) For any given path of the Brownian motion, $a(\tau_1,\ldots,\tau_k)\subsetneq a(\tau_1,\ldots,\tau_{k-1})$. Moreover, for all $i\neq i'$ and for all $t>0$ we have $P(q_i(t)=q_{i'}(t))=0$. Hence, $x_{k+1}(t|\tau_1,\ldots,\tau_k)=$

[53] Observe that the mapping ψ_{I-j} is a mere relabeling of the dimensions of the Brownian motion.

$E\left[\max_{i\in a(\tau_1,\ldots,\tau_k)} q_i(t)\right]$
$< E\left[\max_{i\in a(\tau_1,\ldots,\tau_{k-1})} q_i(t)\right] = x_k(t|\tau_1,\ldots,\tau_{k-1})$, as claimed.

B Extensions

B.1 Discrete time limits

Here we show that our main results – the dispersion of hiring times, and the indeterminate order in which employers exit the market – are not an artifact of the continuous time formulation we adopt. To do so, we derive the same results as the limit of discrete time games.

Let $(\mathcal{T}^n)_n$ be a sequence of partitions of $[0, T]$ into intervals $[\tau_k^n, \tau_{k+1}^n)$ and the element $\{T\}$, such that l_n, defined as the length of the widest interval in element $\mathcal{T}^n$, converges to 0 as n approaches infinity. For each n define $\tilde{\tau}_+^n = \min\{\tau_k^n : \tau_k^n \geq t_1\}$ and $\tilde{\tau}_-^n = \max\{\tau_k^n : \tau_k^n < t_1\}$.

We derive the set of subgame perfect equilibria for each n. If one employer has exited previously, the remaining employer finds it optimal to stay in the game until T. Hence, we can summarize each employer's strategy by noting the first point in time at which this employer exits if both employers are still in the game. If the more desirable employer 1 would exit at t_k^n, then, as long as $x_1(t_{k-1}^n) > x_2(T|t_k^n)$, the less desirable employer 2 finds it optimal to preempt employer 1 by exiting at t_{k-1}^n. Similarly, if the less desirable employer 2 finds it optimal to exit at t_{k-1}^n, then, as long as $x_1(t_{k-1}^n) > x_2(T|t_{k-1}^n)$, the more desirable employer 1 finds it optimal to preempt employer 2 by exiting at t_{k-1}^n too. For all n with $\tilde{\tau}_+^n > t_1$, backwards induction thus implies that in equilibrium either (i) employer 2 exits at $\tilde{\tau}_-^n$ with employer 1 exiting at T, or (ii) that employer 1 exits at $\tilde{\tau}_+^n$ with employer 2 exiting at T. Case (i) applies if $x_1(\tilde{\tau}_-^n) > x_2(T|\tilde{\tau}_+^n)$. This, in turn, holds if t_1 is located at the lower end in the interval $[\tilde{\tau}_-^n, \tilde{\tau}_+^n)$. Case (ii) applies if the opposite inequality holds, which holds if t_1 is located at the upper end in the interval $[\tilde{\tau}_-^n, \tilde{\tau}_+^n)$. (If $\tilde{\tau}_+^n = t_1$, then there are two equilibria; one employer exits at t_1, and the other exits at T.)

Clearly, both $\tilde{\tau}_-^n$ and $\tilde{\tau}_+^n$ converge to t_1, and in each subgame equilibrium one employer exits at T. This shows that in the limit of the discrete time games the hiring times are given as in the continuous formulation. Additionally, continuity of the functions $x_1(\cdot)$ and $x_2(T|\cdot)$ allows us to choose the sequence $(\mathcal{T}^n)_n$ both such that $x_1(\tilde{\tau}_-^n) > x_2(T|\tilde{\tau}_+^n)$ holds for each n, or that the opposite inequality holds for each n. Thus, both orders of employers exiting the market can be sustained as a limit of discrete time games.

Finally, note that the difference in the expected productivity of the students that employers 1 and 2 are matched to approaches zero from below, and does so monotonically if the sequence of partitions $(\mathcal{T}^n)_n$ is nested. Hence, the more frequent the opportunities for employers to hire students, the smaller the difference in expected worker productivity across employers.

B.2 Allowing for student rejections

Here, we study an extension of the model to the case in which students can reject offers. We find that the qualitative features of the equilibrium depend on the parameters. We show that the model with student rejections approaches the model without rejections as students' utility from the outside option decreases (while their valuations of employers remain constant). Accordingly, the model in the main text can be interpreted as one of *elite* entry level labor markets, in which each position is preferred to the outside option.

Assumptions All students assign utility u_1 to a match with employer 1, u_2 to a match with employer 2, and u_0 to being unemployed, with $u_1 \geq u_2 \geq u_0$. Without loss of generality, we set $u_1 = 1$ and $u_2 = 0$. Students have the same information about the evolution of student skills as employers. That is, the process according to which student skills evolve is common knowledge, but students do not possess any information about the current realization of students' skills, not even their own. Once a student receives an offer, however, he can deduce, that he currently is the most highly skilled available student. We use $p^i(t, t')$ to denote the probability that the student who is ranked first at time t will be ranked i at time $t' > t$. It is clear that $p^i(t, t')$ is continuous for all i, and first-order stochastically increasing in t for any fixed t'.[54]

An employer can make an offer to the best available student by playing an action $e \in E$, as in Section 3. Whenever an employer j does so, time is halted, the best available student learns the identity of the employer making the offer, and must decide immediately whether or not to accept this employer's offer. If he accepts, he and employer j are matched, and exit the game. Otherwise both players remain in the game. Hence, offers are exploding. We assume that in case of indifference, students accept an offer. Students do not observe offers that are made to other students if they are rejected. All other past actions of all agents are common knowledge.

[54] That is, $p^1(t, t')$ and $p^1(t, t') + p^2(t, t')$ are both increasing in t.

Note that accepting an offer from employer 1 is a weakly dominant strategy for any student, and so is accepting the best offer received at time T. Hence, we can represent a student's strategy as a mapping $\mathcal{H} \to \{a, r\}$ where a and r are the actions of accepting and rejecting an offer by employer 2, respectively. Additionally, an employer's strategy of making an offer to the currently second best available student at any time $t < T$ is strictly dominated by the strategy of hiring the best available student at T. Therefore, we lose no generality by assuming that employer's actions at each point in time t are whether or not to make an offer to the currently best available student. For tractability we only consider equilibria that are symmetric in the sense that all students play the same strategy. For notational convenience, instead of writing "the currently best available student", we simply write "the student".

Analysis Suppose that the student knows that the next point in time at which employer 1 will make an offer to the then best available student is t. We can then derive the last point in time $\tau(t)$ at which the student would accept an offer from employer 2. From accepting employer 2's offer, a student receives utility u_2. Comparing this to the expected utility from rejecting the offer, we obtain

$$\tau(t) \;\; = \;\; \max\{t' : p^1(t', t)u_1 + p^2(t', t)u_2 + (1 - p^1(t', t) - p^2(t', t))u_0 \leq u_2\}$$

We characterize the equilibria of this game depending on the value of $\tau(T)$. As in the case without student rejections, we define t_1 by $x_1(t_1) = x_2(T|t_1)$. Additionally, we define $\bar{t}$ by $x_1(\bar{t}) = x_2(T|T)$ (see figure B.4). We distinguish between three cases, depending on the location of $\tau(T)$ relative to t_1 and $\bar{t}$.

Case 1 The case $\tau(t) < \bar{t}$ obtains if u_0 is high, so that students have a strong preference for employer 1 over employer 2 relative to their fear of unemployment. Hiring before $\bar{t}$ is not attractive for either employer: postponing hiring until T yields a higher expected payoff even for the employer who will match to the second best student at T. Employer 2's offer to the best student will be rejected if made at any time $t \in (\tau(T), T)$. Hence employer 1 faces no competition in this time interval, and thus has no reason to make an offer. Thus, the student's assumption that no offer will be made in the interval $(\tau(T), T)$ is correct. Hence, there is an equilibrium without unraveling in which employers will assortatively match to students at T.

Case 2 Next, we consider the case $\bar{t} \le \tau(T) \le t_1$, which obtains if u_0 is in an intermediate range such that students have somewhat weaker preferences for employer 1 over employer 2 relative to their fear of unemployment. Employer 2 prefers to make an offer to a student as late as possible. But he also strives to avoid competition by employer 1. Hence, employer 2 will find it optimal to make an offer to the best student at $\tau(T)$, and this student will accept. Employer 1 faces no competition in the time interval $(\tau(T), T)$, and hence will not make any offer during this interval. Moreover, making an offer at or before $\tau(T)$ is a weakly dominated strategy for employer 1. This justifies the student's assumption that no offer will be made in the interval $(\tau(T), T)$. Notably, in this equilibrium, the students' ability to reject offers exacerbates unraveling relative to the case without student rejections.

Case 3 Finally, we study the case $t_1 < \tau(T)$, which is most similar to the case without student rejections. A pure strategy subgame perfect equilibrium always exists, since giving the students the opportunity to reject offers essentially renders the model into a discrete time game, through the following argument. By assumption, at time $\tau(T)$, the last point in time at which the student would accept employer 2's offer, exiting beats staying in the game for both employers. Thus employer 1 will make an offer to the student at $\tau(T)$. Consequently, the student will not accept an offer from employer 2 sufficiently shortly before $\tau(T)$, since he knows that he will receive an offer from employer 1 at time $\tau(T)$. There will, however, be a latest time before $\tau(T)$, denoted $\tau^2(T)$, at which the student would accept employer 2's offer. If that point in time is is again strictly later than t_1^*, exiting at $\tau^2(T)$ beats waiting for both employers, and hence, employer 1 will make an offer to the student at that point in time, and we can iterate the previous argument. Thus, the students' strategy essentially renders the game into a discrete time game between employers, since employer 2 can only possibly exit at times $\tau^n(T)$. In equilibrium, the first exit occurs at one of the two grid points that are closest to t_1. Which one it is depends on whether exiting at the grid point just before t_1^* is better or worse for employer 2 than being preempted by employer 1 at the first grid point after t_1^*. As always, the second exit occurs at T. Moreover, the discrete time grid that emerges becomes increasingly fine as the utility u_0 from the outside option decreases, since the student becomes increasingly unwilling to wait for a any given amount of time in hope for an offer from employer 1. Hence, as u_0 approaches $-\infty$, the time of first exit approaches t_1. The order of exit in the limit is indeterminate in the following sense: For each $\underline{u} \in \mathbb{R}$ a value $u_0' < \underline{u}$ can be found such that employer 1 is first to exit, and another value $u_0'' < \underline{u}$ can be found such that employer 2 is first to exit.

To formally describe the equilibrium in this case, we set $\tau^0(T) = T$. Inductively, we then define $\tau^n(T) = \tau(\tau^{n-1}(T))$ as the last point in time strictly before $\tau(\tau^{n-1}(T))$ at which the student is willing to accept employer 2's offer if he knows that he will receive an offer from employer 1 at time $\tau^{n-1}(T)$ if he is still the best available student at that time. We call the points in time $\{\tau^n(T)|n \in \mathbb{N}\}$ *threat times*. Define $\tau_+^* = \min\{\tau^n(T) : \tau^n(T) \geq t_1, n \in \mathbb{N}\}$, and $\tau_-^* = \tau(\tau_+^*)$

We define the players' strategies as follows:

- The student accepts an offer from employer 2 at all threat times $\tau^n(T) \geq \tau_-^*$, and rejects it otherwise.
- Employer 1 plays e_1 at each threat time $\tau^n(T) \geq \tau_+^*$, and plays s otherwise.
- Employer 2 plays e_1 at each threat time $\tau^n(T) \geq \tau_+^*$, additionally plays e_1 at threat time τ_-^* if $x_1(\tau_-^*) \geq x_2(T|\tau_+^*)$, and plays s otherwise.

We show that this pure strategy profile is subgame perfect Nash equilibrium. Our argument relies on the fact that for any $t > 0$ the interval $[t, T]$ contains a finite number of threat times. To see this, suppose that $\tilde{t} > 0$ is an accumulation point of threat times. This means that for each $\epsilon > 0$ we have

$$p^1(\tilde{t}, \tilde{t} + \epsilon)u_1 + p^2(\tilde{t}, \hat{t} + \epsilon)u_2 + \big(1 - p^1(\tilde{t}, \tilde{t} + \epsilon) - p^2(\tilde{t}, \tilde{t} + \epsilon)\big)u_0 > u_2$$

Note that $p_1(\tilde{t}, \tilde{t}) = 1$. Therefore, the left hand side of the above expression equals u_1 when $\epsilon = 0$, which strictly exceeds u_2. This contradicts the fact that $(p_1(t', t), p_2(t', t))$ is continuous in (t', t).

We now show that no player has an incentive to unilaterally deviate. Clearly, making an offer at any time $t \in (\tau^1(T), T)$ is suboptimal for employer 1, since at any such point in time an offer by employer 2 would be rejected, so that this employer is no serious competitor at such a time. If $\tau^1(T) > t_1$, then employer 2 will make an offer at that time. Because in this case, $x_1(\tau^1(T)) > x_2(T|\tau^1(T))$, employer 1 will also make an offer at that time. Therefore, for any point in time $t \in (\tau^2(t), \tau^1(T))$, rejecting employer 2's offer is rational for the student, and hence employer 1 will not make an offer at any such point in time. But

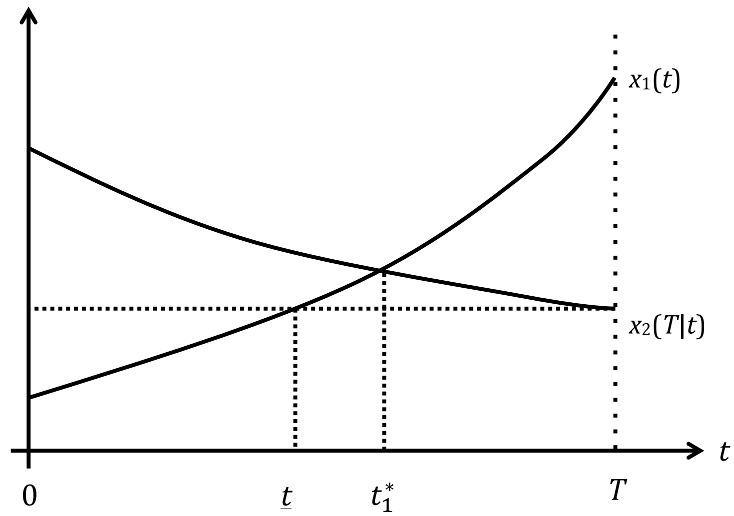

Figure B.4: The model with two employers when students can reject offers. The first exit never occurs before $\underline{t}$, but may occur between $\underline{t}$ and t_1^*.

since the student will accept employer 2's offer if he receives it at time $\tau^2(T)$, as long as $x_1(\tau^2(T)) > x_2(T|\tau^2(T))$, employer 1 will also make an offer at that point in time. When the first offer will be made, an by whom it will be made now depends on the exact location of τ_+^* and τ_-^*. By definition of τ_+^*, if both employers are still in the game at that point in time, both employers will play e_1, employer 1 will exit, and employer 2 will earn expected payoff $x_2(T|\tau_+^*)$. Instead, by preempting employer 1 and playing e_1 at τ_-^*, he can instead obtain expected payoff $x_1(\tau_-^*)$.

In conclusion, depending on the precise value of u_0, one of the following two outcomes is consistent with a subgame perfect pure strategy equilibrium profile:

- Employer 2 plays e_1 at τ_-^*, the student accepts, and employer 1 exits at T.
- Both employer 1 and employer 2 play e_1 at τ_+^*, the student employer 1's offer, and employer 2 exits at T.

Finally, it is clear that as u_0 approaches $-\infty$, the distance between any two threat times approaches 0. In particular, $|\tau_-^* - t_1|$ and $|\tau_+^* - t_1|$ approach 0. In this sense, the model with student rejections converges to the model without student rejections.

B.3 Fully observable student skill levels

Our model is tractable because employers cannot condition their strategies on the current realization of students' skills – they possess no information that would enable them to do so. In this section we relax this assumption. Employers have full knowledge of the current realization of all students' skill levels at each point in time. We formally derive a subgame perfect equilibrium in the case of two employers. We find that hirings are still dispersed in time, and any employer faces the same ex ante expected payoff from hiring at any of these points in time in equilibrium. By arguments that parallel those in Section 4.6, continuous changes in the speed of arrival of information have no effect on equilibrium payoffs, and the mechanism that is employed by a centralized clearinghouse has no effect on equilibrium behavior.

Setup Each students' skill level is given by an independent standard Brownian motion (see Appendix A.7). Both employers can, at any time, make an offer to any student i of their choosing by playing action e_i, or they can choose to play action s and remain in the game. Students must accept any offer they receive, except in the case that they receive multiple offers. In this case, the more attractive employer 1 is assigned the student. For simplicity, we assume that an employer's payoff from failing to hire a student is $-\infty$.

Analysis Let $x_1(t,q)$ denote the skill level that an employer can expect at t if he hires the best student at t, given that the vector of student skill level at that time is q. By the martingale property of the Brownian motion it is immediate that

$$x_1(t,q) = \max_i q_i$$

Once a student has been hired, the best response of the remaining employer is to wait until T and hire the best available student then. Let t be the point in time at which the first student was hired. Let

$$x_2(t,q) = E_t\Big[\max\big\{q_i(T) : 1 \le i \le I, i \ne \overline{\text{argmax}}_i q_i(t)\big\}\Big]$$

Here, the function $\overline{\text{argmax}}$ returns the lowest-index maximizer of its argument. $x_2(t,q)$ is the expected value of this strategy conditional on the information available at t. It depends on the vector of skill levels q that prevailed at t, and on the distance between t and the start date of employment, T.

As in our baseline model, we would like to define a point in time at which exiting the game yields the same expected payoff as remaining in the game. This will now depend on the realizations of the student skill levels. Specifically, we define $A \subset [0,T] \times \mathrm{R}^I$ as the set of pairs (t,q) such that the expected value from hiring the best student is equal to the expected value from the continuation strategy $x_2(t,q)$,

$$A = \{(t,q) \in [0,T] \times \mathrm{R}^I : x_1(t,q) - x_2(t,q) \geq 0)\}$$

The set A characterizes a subgame perfect equilibrium of the game.[55]

Proposition 7. *Define employers' strategies σ_1 and σ_2 as follows. If no student has yet been hired and $(t,q) \in A$, or if a student has been hired and $t = T$, then $\sigma_j(t,q) = e_{\overline{\mathrm{argmax}}_i q_i}$. Otherwise, $\sigma_j(t,q) = s$. Then, $\sigma = (\sigma_1, \sigma_2)$ is a subgame perfect equilibrium.*

Proof. Suppose employer $-j$ plays strategy σ_{-j}. It is immediate that σ_j is a best response to σ_{-j} if and only if σ_j prescribes waiting with hiring until T if employer $-j$ has already exited. Moreover, by the definition of A, employer j strictly prefers playing s to playing e_1 at any $(t,q) \notin A$, and weakly prefers playing e_1 at any $(t,q) \in A$. Consequently, σ_j is a best response to σ_{-j}. $\square$

Observe that each employer's ex ante expected in this equilibrium is the same, and that the points in time at which students are hired are dispersed in time, with the last student being hired at T. This parallels parts (i) and (ii) of Corollary 1 in the main text. Moreover, it is apparent that continuous changes in the speed of arrival of information have no effect on equilibrium payoffs. It is also apparent that at most one student will be hired at any given point in time, so that the mechanism that is employed by a centralized clearinghouse will have no effect on equilibrium behavior.

[55] Observe that the boundary of A is a one-dimensional smooth submanifold of $[0,T] \times \mathrm{R}^I$. By setting x_3 as the conditional expectation of the maximal student skill at the point in time at which the process hits A, we could, in principle, generalize this analysis to 3 employers, and use induction to extend the analysis to n employers. Doing so requires one to prove a theorem about the smoothness of hitting distributions of I-dimensional Brownian motions on submanifolds of $[0,T] \times \mathrm{R}^I$.

C Data Sourcing and Encoding

We code the timing data as follows. For each of the twelve regional federal court of appeals circuits[56] we made note of the exact time and date of every blog entry that included information on hiring and interviewing. We categorized these entries in two dimensions. First, we noted whether claims were being made about (i) students still in law school, (ii) individuals who had previously graduated from law school, or (iii) whether it was unclear. Second, we classified entries as claims that a judge (a) was reviewing applications, (b) was making telephone calls or sending emails to schedule interview times, (c) was interviewing, (d) had made a hire, or (e) had finished the hiring process. We adhered to the following four rules while categorizing our data. 1. Unless blog entries were subsequently claimed to be false or corrected, we assumed every blog entry that was phrased as a statement rather than a question to be evidence that a hiring event occurred. 2. Clear confirmations of events were treated as repeated data points and were not included in our analysis. (For example, while we recorded the entry "Bea called to set up interviews" on September 8, 2008 at 8:48 pm, we did not record the subsequent entry "I can confirm that Bea definitely set up at least two interviews today." on September 8, 2008 at 11:35 pm. Whereas, both of the following entries were treated as unique data points: "Gorsuch has hired two for the current term..." on July 7, 2008 at 8:24 am and "Gorsuch has hired a ... clerk from Harvard." on July 17, 2008 at 3:46 pm.) 3. We did not include very late entries which occurred considerably after the majority of activity had ceased since this was most likely relevant to the 2010-2011 season. 4. We recorded the date of each hire, interview, review, etc. as the time at which the event was recorded on the blog. We did not take into account any comments about when the event occurred (such as "Interviewed with Briscoe last week..."). One exception to this rule is that there were a number of entries that explicitly stated that calls were made at, or a few minutes after, 12:00 pm on September 8th. These blog entries were mostly all posted within a few hours of 12:00 pm and were recorded as 12:00 pm rather than the precise time at which they were posted.

The 168 judges represented in Figure 1 are those for whom we observe statements that they have hired a clerk or have finished the hiring process. Our data contains entries about

[56]The number of blog entries for the federal circuit court of appeals is very limited, most likely because many federal circuit judges do not hire current law students. For this reason, we exclude these entries from our analysis.

29 additional judges that contain neither type of information. Moreover, some data points may concern the hiring of students who have already graduated, and to whom the Hiring Plan does not apply. For the vast majority of the blog entries in our data it was not possible to determine whether they were referring to a current or graduated law student. The fact that the hiring plan was officially abandoned in November 2013 because fewer and fewer judges adhered to it makes us confident that a significant number of our data points refer to current law students rather than graduates.